Math 117 - Spring 2023 - Common Final Exam, version A Solutions

1. Alice is deciding between two bids for a project. Both have fixed costs for materials and an hourly rate for labor. Company A has bid \$2000 for materials and \$60 per hour for labor. Company B has bid \$1200 for materials and \$80 per hour for labor.

   (a) (4 points) Write a linear function for each company's bid as a function of t, the amount of labor required (in hours).

   Solution and rubric:

1 pt	At least one bid as some sort of linear function of t is given
1 pt	Evidence for correct interpretation of slope
1 pt	Evidence for correct interpretation of intercept
1 pt	All details are correct

   Company A: $C = 2000 + 60t$ Company B: $C = 1200 + 80t$

   (b) (4 points) At what amount of labor time are the two bids equal? Give correct units for your answer.

   Solution and rubric:

2 pt	Set both cost function equal
1 pt	Progress with algebra, may be some error
1 pt	correct answer with units

   Solving $2000 + 60t = 1200 + 80t$ gives 40 hours.

   (c) (3 points) For what amounts of labor time is Company A's bid cheaper than company B's bid? Give your answer using interval notation or inequalities, or in a sentence.

   Solution and rubric:

1 pt	Inequality involving answer from (b)
2 pt	correct direction

   The correct answer is $t > 40$. Award full credit for answers consistent with the answer from (b).

2. Let $P = f(t)$ be the population (in millions) of a country t years after 2020.

   (a) (3 points) In a sentence with correct units, explain the meaning of $3.4 = f(4)$.

   Solution and rubric:

   The country's population is 3.4 million people in 2024.

   (b) (3 points) What are the units of the average rate of change $\frac{\Delta P}{\Delta t}$ for this function?

   (b) **millions of people pe**

   (c) (3 points) In a sentence with correct units, explain the meaning of $f^{-1}(7) = 8$.

Solution and rubric:
A population of 7 million people will occur in 2028.

Solution and rubric:

For parts (a) and (c):

1 pt	A sentence relating the quantities with at least one correct unit applied to a quantity.
2 pts	Correct sentence with units for both quantities.

For part (b):

1 pt	A units expression using at least one of "millions of people" and "year"
2 pts	Correct units for RoC with "per" or "/"

3. The height (in feet) of an object after t seconds is given by

$$h(t) = -16(t-2)^2 + 75.$$

(a) (4 points) Determine the maximum height of the ball and the time when that maximum occurs. Include correct units in your answer.

Solution and rubric:

1 pt	At least one coordinate of vertex correctly interpreted
1 pt	Both coordinates of vertex correctly interpreted.
1 pt	At least one correct unit in answer.
1 pt	Both correct units in answer.

The ball reaches a maximum height of 75 feet after 2 seconds.

(b) (2 points) Give the equation of the line which is the axis of symmetry of the graph of $y = h(t)$.

Solution and rubric:

1 pt	Correct x coordinate for the line, but not in the form $x = k$
1 pt	Correct equation

$x = 2$

(c) (4 points) Give the formula for the ball's height in standard form: $h(t) = at^2 + bt + c$.

Solution and rubric:

$h(t) = -16t^2 + 64t + 11$

4. The table below gives some values of a function f and average rates of change $\frac{\Delta f}{\Delta x}$.

x	2	6	10	14
$f(x)$	20	18	14	2
$\frac{\Delta f}{\Delta x}$		$-\frac{1}{2}$	?	-3

(a) (4 points) Find the missing rate of change in the table.

Solution and rubric:

1 pt	some ratio computed but may be $\frac{\Delta x}{\Delta f}$
2 pt	correct ratio computed but might have incorrect endpoints or other error
1 pt	correct computation

$\frac{\Delta f}{\Delta x} = \frac{14-18}{10-6} = -1$

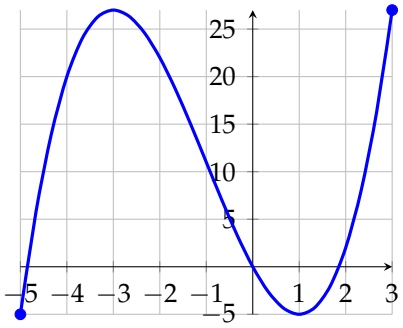
1 pt	answer at least references rate of change
1 pt	answer at least references rate of change of rate of change
1 pt	answer is correct

(b) (3 points) Is f concave up or concave down? Explain your answer in a sentence using the average rates of change in the table.

Solution and rubric:

f is concave down. We know this bcause the rates of change are decreasing.

5. The graph $y = f(x)$ is given below. Use it to answer the following questions.



(a) (2 points) Estimate $f(1)$.

(a) **-5**

(b) (3 points) Estimate all solutions to $f(x) = 10$.

(b) $x \approx -4.5, -1, 2.5$

Solution and rubric:
1 pt per solution

(c) (4 points) Estimate the average rate of change $\frac{\Delta f}{\Delta x}$ over the interval $-4 < x < 1$.

Solution and rubric:

1 pt	some ratio computed but may be $\frac{\Delta x}{\Delta f}$
2 pt	correct ratio computed but might have incorrect endpoints or other error
1 pt	correct computation

$$\frac{\Delta f}{\Delta x} \approx \frac{-5 - 20}{1 - (-4)} = -5$$

Solution and rubric:
For parts (d) and (e):

1 pt	at least one correct choice
1 pt	both correct choices
1 pt	no incorrect choices

(d) (3 points) Over which intervals is f increasing? Circle all correct choices.
A. $-5 < x < -3$ B. $-3 < x < -2$ C. $-1 < x < 2$ **D.** $2 < x < 3$

(e) (3 points) Over which intervals is f concave up? Circle all correct choices.
A. $-5 < x < -4$ B. $-4 < x < -2$ **C.** $-1 < x < 2$ **D.** $2 < x < 3$

6. (5 points) Let $b(t) = t + 2$ and $r(t) = 3t^3$. Find a formula for the composition $b\big(r(t)\big)$.

Solution and rubric:

2 pt	some composition, may be the wrong way
3 pt	correct formula for composition

$b\big(r(t)\big) = 3t^3 + 2$

7. Suppose that $(4,7)$ is a point on the graph of f. Give a point on the graph of each of the following functions defined in terms of f.

Solution and rubric:

For these parts:	1 pt	pogress toward correct y coordinate
	1 pt	pogress toward correct x coordinate
	1 pt	no algebra errors; correct answer

(a) (3 points) $g(x) = 3f(x) - 2$

(a) $(4, 19)$

(b) (3 points) $h(x) = -f\left(\frac{1}{2}x\right)$

(b) $(8, -7)$

(c) (3 points) $h(x) = f\left(3(x+1)\right) - 5$

(c) $(\frac{1}{3}, 2)$

8. (6 points) Let $h(t) = \frac{3}{t-2}$. Find a formula for the inverse function h^{-1}.

Solution and rubric:

Students may swap independent and dependent variables or not.

2 pt	evidence of solving for independent variable
2 pt	correct solution
1 pt	correct answer with consistent variables in the formula

$h^{-1}(y) = \frac{3}{y} + 2$

9. (4 points) Find the domain of $g(x) = \dfrac{8}{\sqrt{x+7}}$. You may express your answer using interval notation, inequalities, or in a sentence.

Solution and rubric:

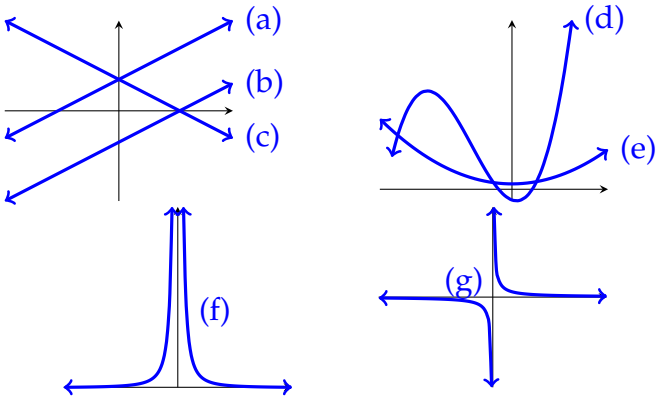
1 pt	evidence of avoiding division by zero or square root of negative
2 pt	correct direction of inequality and endpoint
1 pt	do not include $x = -7$

Domain: $x > -7$

10. (5 points) Match each of the following functions with a graph by writing the letter in the blank. The horizontal and vertical scales on the graphs may not be equal. Not all letters will be used.

Solution and rubric:

1 pt per correct choice



__**g**__ $y = \dfrac{10}{x}$

__**d**__ $y = x^3 + 10x^2 - 8x - 20$

__**e**__ $y = x^2 + 10$

__**a**__ $y = 3x + 8$

__**b**__ $y = 3x - 8$

11. Consider the rational function $y = r(x) = \dfrac{(x-2)(3x+1)}{(x-2)(x-4)}$.

(a) (3 points) Give the (x, y) coordinates of any holes in the graph of r.

Solution and rubric:

2 pt	correct x coordinate
1 pt	correct y coordinate

r has a hole at $\left(2, -\frac{7}{2}\right)$

(b) (3 points) Give the equation of any horizontal asymptotes of r.

Solution and rubric:

2 pt	correct value for asymptote
1 pt	correctly expressed equation $y = 3$

12. Consider the rational function $y = g(x) = \dfrac{-3(x+2)}{(x+5)\left(x-\frac{2}{3}\right)}$.

(a) (3 points) Give the equation of any vertical asymptotes of g.

Solution and rubric:

1 pt	at least one correct value for asymptote
1 pt	both correct values for asymptote
1 pt	correctly expressed equations $x = -5$ and $x = \frac{2}{3}$

(b) (3 points) Give the x coordinate for any x-intercept of the graph of g.

Solution and rubric:

2 pt	correct intercept
1 pt	nothing else added

r has an x intercept of -2

(c) (3 points) Fill in the blanks with the correct end behavior of g.

Solution and rubric:

2 pt	at least one correct limit
1 pt	both correct

$$\lim_{x\to\infty} g(x) = \underline{\quad \mathbf{0} \quad} \text{ and } \lim_{x\to-\infty} g(x) = \underline{\quad \mathbf{0} \quad}$$

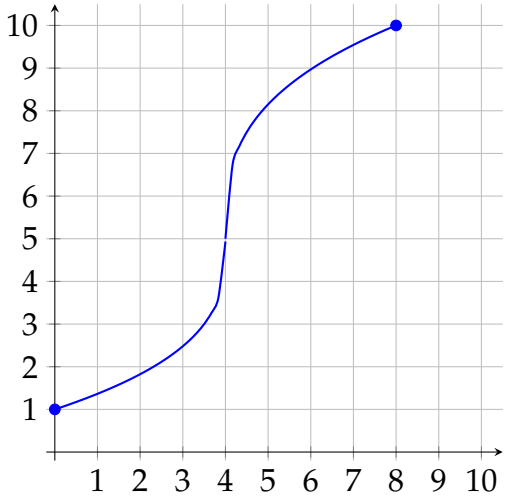
13. (6 points) On the following set of axes, draw the graph of a function with these properties:

- The domain of the function is $0 \leq x \leq 8$.
- The range of the function is $1 \leq y \leq 10$.
- The function is increasing over its whole domain.
- The function is concave up for $0 < x < 4$ and concave down for $4 < x < 8$.

Solution and rubric:

1 pt	correct domain
1 pt	correct range
1 pt	increasing
1 pt	concave up on $0 < x < 4$
1 pt	concave down on $4 < x < 8$

The graph below is one possible solution.



14. (8 points) Write a formula for a degree three polyomial with zeros at $x = -3$, $x = 4$, and $x = 5$, and which passes through the point $(0, 6)$.

Solution and rubric:

1 pt	evidence of using factored form
1 pt	at least one zero correctly represented
2 pt	all zeros correctly represented
1 pt	attempt to solve for leading coefficient
2 pt	set up with $(0, 12)$ to solve for leading coefficient
1 pt	correct formula

$y = \frac{1}{10}(x+3)(x-4)(x-5)$

15. Consider the function

$$f(x) = \begin{cases} -2x + 2 & \text{for } x \leq 0 \\ x^2 + 2 & \text{for } x > 0 \end{cases}.$$

(a) (3 points) Compute $f(4)$.

Solution and rubric:

1 pt	correctly plugging into formula, but may incorrectly use both formulas $f(4) = (4)^2 + 2 = 18$.
1 pt	Use only one formula
1 pt	use only one formula, and no errors in computation

(b) (3 points) Find all solutions to $f(x) = 18$.

Solution and rubric:

1 pt	set up at least one equation to solve for x
1 pt	correct solution(s) of equation(s)
1 pt	give only the solutions in the correct part of the domain, $x = -8$ and $x = 4$

Solve $-2x + 2 = 18$ to get $x = -8$ and solve $x^2 + 2 = 18$ to get $x = 4$. Reject the negative root,

Solution and rubric:

Points for the following parts are all-or-nothing.

(c) (2 points) What is the domain of f?

(c) **all real numbers**

(d) (2 points) What is the range of f?

(d) $y \geq 2$